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FCNC top quark decays in extra dimensions

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Abstract. The flavor changing neutral top quark decay $t \to cX$ is computed, where X is a neutral standard model particle, in an extended model with a single extra dimension. The cases for the photon, $X = \gamma$, and a standard model Higgs boson, $X = H$, are analyzed in detail in a non-linear R_ξ gauge. We find that the branching ratios can be enhanced by the dynamics originating in the extra dimension. In the limit where $1/R \gg m_t$, we have found $\mathrm{Br}(t \to c\gamma) \simeq 10^{-10}$ for $1/R = 0.5$ TeV. For the decay $t \to cH$, we have found $\mathrm{Br}(t \to cH) \simeq 10^{-10}$ for a low Higgs mass value. The branching ratios go to zero when $1/R \to \infty$.

1 Introduction

Flavor changing neutral currents (FCNC) are very suppressed in the standard model (SM): there are no tree level contributions and at one loop level the charged currents operate by the Glashow–Iliopoulos–Maiani (GIM) mechanism. The branching ratio for top quark FCNC decays into charm quarks are of the order of 10^{-11} for $t \to cg$ and 10^{-13} for $t \to c\gamma, (Z)$ in the framework of the SM [1–6]. This suppression can be traced back to the loop amplitudes: they are controlled by down-type quarks, mainly by the bottom quark, resulting in a m_b^4/M_W^4 factor, which can be compared with the enhancement factor that appears in the $b \to s\gamma$ process, in which the top quark mass m_t is involved instead of m_b in this factor. This fourth power mass ratio is generated by the GIM mechanism and is responsible for the suppression beyond naive expectations based on dimensional analysis, power counting and Cabibbo–Kobayashi–Maskawa (CKM) matrix elements involved. The top quark decay into the SM Higgs boson is even more suppressed [1–6]: $\mathrm{Br}(t \to cH) \sim 10^{-13}$–$10^{-15}$ for $M_Z \leq M_H \leq 2M_W$. These rates are far below the reach of any foreseen high luminosity collider in the future. The highest FCNC top quark rate in the SM is $t \to cg$, but this value is still six orders of magnitude below the possibility of observation at the LHC.

The discovery of these FCNC effects would be a hint of new physics because of the large suppression in the SM. These FCNC decay modes can be strongly enhanced in scenarios beyond the SM; some of them could even be observed at the LHC or ILC. New physics effects in extended Higgs sector models, SUSY and left–right symmetric models were studied in [1–21]. For example, in various SUSY scenarios the branching ratios may reach the value

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10^{-5} for the decay $t \to cg$. Also, virtual effects of a Z' gauge boson on these rare top quark decays were studied. The decay $t \to c\gamma$ has been analyzed in [22], and it has been shown that $\mathrm{Br}(t \to c\gamma)$ is at the 10^{-6} level in topcolor assisted technicolor models for $m_{Z'} = 1$ TeV, which would allow for the detection of this process at future colliders.

On the other hand, the use of effective Lagrangians in parameterizing physics beyond the SM has been studied extensively in FCNC top quark couplings and decays [23–29]. This formalism generates a model-independent parameterization of any new physics characterized by higher dimension operators. Under this approach, several FCNC transitions have also significantly been constrained: $t \to c\gamma$ [30, 31], $t \to cg$ [31, 32], $l_i \to l_j\gamma$ [33] and $H \to l_i l_j$ [34].

New physics effects have also been introduced in models with large extra dimensions (ED) [35–38]. In recent years, these models have been a major source of inspiration for physics beyond SM in ongoing research. In these scenarios the four dimensional SM emerges as the low-energy effective theory of models living in more than four dimensions, where these extra dimensions are orbifolded. The infinite towers of Kaluza–Klein (KK) modes present are remnants of the extended dimensional dynamics at low energies. The size of the extra dimensions can be unexpectedly large, with $1/R$ at the scale of a few TeV without contradicting the present experimental data [39–48]. Then, if these KK modes are light enough, they could be produced in the near future at the next generation of colliders. Scenarios where all the SM fields, fermions as well as bosons, propagate in the bulk are known as models with "universal extra dimensions" [49–52]. In these theories the number of KK modes is conserved at each elementary vertex and the coupling of any excited KK mode to two zero modes is prohibited. Then the constraints on the size of the extra dimensions obtained from the SM precision measure-

ments are less stringent than in the case in which there is no conservation of the KK particles (non-universal extra dimensions).

The impact of the new physics coming from UED models has been widely studied and constraints on the parameter $1/R$ have been obtained. Analysis of the precision electroweak observables led to the lower bound $1/R \gtrsim 700$–$800\,\mathrm{GeV}$ for a light Higgs boson mass and to $1/R \gtrsim 300$–$400\,\mathrm{GeV}$ for a heavy Higgs boson mass [53, 54]. On the other hand, using the process $b \to s\gamma$, the resulting bound on the inverse compactification radius is $1/R \gtrsim 250\,\mathrm{GeV}$ [55–57]. Moreover, a recent analysis making use of the exclusive branching ratio $B \to K^*\gamma$ shows that under conservative assumptions $1/R \gtrsim 250\,\mathrm{GeV}$ [58]. Also, from the inclusive radiative $\bar{B} \to X_s\gamma$ decay, a lower bound of $1/R \gtrsim 600\,\mathrm{GeV}$ at 95% C.L. can be obtained, and this is independent of the Higgs boson mass value [59]. Contributions from UED models have been considered in several FCNC processes, and [60] has found that the processes $K_{\mathrm{L}} \to \pi^0 e^+ e^-$, ΔM_s, $K^+ \to \pi^+\nu\bar{\nu}$, $K_{\mathrm{L}} \to \pi^0\nu\bar{\nu}$, $B_{d,s} \to X_{d,s}\nu\bar{\nu}$, $K_{\mathrm{L}} \to \mu^+\mu^-$, $B_{d,s} \to \mu^+\mu^-$, $B \to X_s\mu^+\mu^-$ for $1/R \approx 300\,\mathrm{GeV}$ are enhanced relative to the SM expectation and the processes $B \to X_s\gamma$, $B \to X_s g$, ϵ'/ϵ are suppressed with respect to the SM. In general, the present data on the FCNC processes are consistent with $1/R \gtrsim 300\,\mathrm{GeV}$ [60,61]. Exclusive $B \to K^*l^+l^-$, $B \to K^*\nu\bar{\nu}$ and B_s decays [58,62] have been studied in the framework of the UED scenario and also rare semileptonic Λ_b decays [63,64].

In this paper we study the FCNC decays of the top quark in a universal extra dimension theory in which all the SM fields live in five dimensions. In particular, we compute the $t \to c\gamma$ and $t \to cH$ decay modes in a non-linear R_ξ. This gauge has the advantage of a reduced number of Feynman diagrams as well as simplified Ward identities. These facts facilitate and clarify the calculation.

The paper is organized as follows. In Sect. 2 we present the general framework for the five dimensional Lagrangian and derive the corresponding four dimensional Lagrangian and Feynman rules. In Sects. 3 and 4 we compute the decay modes $t \to c\gamma$ and $t \to cH$, respectively, and discuss the hypothesis implicit in the calculation. Finally, in Sect. 5 we present some conclusions. In the appendix we show the terms in the Lagrangian that are important for the Feynman rules in our calculation.

2 The model

We begin by presenting the SM Lagrangian in five dimensions; let $x = 0, 1, 2, 3$ be the normal coordinates and $x^4 = y$ the fifth one. The fifth extra dimension is compactified on the orbifold S^1/Z_2, an orbifold of size R, which is the compactification radius. We consider a generalization of the SM in which the fermions, the gauge bosons and the Higgs doublet propagate in the five dimensions. The Lagrangian L can be written as

$$L = \int \mathrm{d}^4x \, \mathrm{d}y \, (\mathcal{L}_A + \mathcal{L}_H + \mathcal{L}_F + \mathcal{L}_Y), \qquad (1)$$

with

$$\mathcal{L}_A = -\frac{1}{4}W^{MNa}W^a_{MN} - \frac{1}{4}B^{MN}B_{MN},$$
$$\mathcal{L}_H = (D_M\Phi)^\dagger D^M\Phi - V(\Phi),$$
$$\mathcal{L}_F = \left[\overline{Q}(\mathrm{i}\Gamma^M D_M)Q + \overline{U}(\mathrm{i}\Gamma^M D_M)U + \overline{D}(\mathrm{i}\Gamma^M D_M)D\right],$$
$$\mathcal{L}_Y = -\overline{Q}\widetilde{Y}_u\Phi^c U - \overline{Q}\widetilde{Y}_d\Phi D + \mathrm{h.c.} \qquad (2)$$

The numbers $M, N = 0, 1, 2, 3, 5$ denote the five dimensional Lorentz indices, $W^a_{MN} = \partial_M W^a_N - \partial_N W^a_M + \widetilde{g}\epsilon^{abc}W^b_M W^c_N$ is the field strength tensor for the $\mathrm{SU}(2)_{\mathrm{L}}$ electroweak gauge group and $B_{MN} = \partial_M B_N - \partial_N B_M$ is that of the $\mathrm{U}(1)_Y$ group. The gauge fields depend on x and y. The covariant derivative is defined as $D_M \equiv \partial_M - \mathrm{i}\widetilde{g}W^a_M T^a - \mathrm{i}\widetilde{g}'B_M Y$, where $\widetilde{g}$ and $\widetilde{g}'$ are the five dimensional gauge couplings constants for the groups $\mathrm{SU}(2)_{\mathrm{L}}$ and $\mathrm{U}(1)_Y$, respectively, and T^a and Y are the corresponding generators. The five dimensional gamma matrices Γ_M are $\Gamma_\mu = \gamma_\mu$ and $\Gamma_4 = \mathrm{i}\gamma_5$ with the metric tensor given by $g_{MN} = (+, -, -, -, -)$. The matter fields Q, D and U are fermionic four components spinors with the same quantum numbers as the corresponding SM fields. To simplify the notation we have suppressed the $\mathrm{SU}(2)$ and color indices. The standard and charge conjugate doublet standard Higgs fields are denoted by $\Phi(x, y)$ and $\Phi^c(x, y) = \mathrm{i}\tau^2\Phi^*(x, y)$; $\widetilde{Y}_{u,d}$ are the Yukawa matrices in the five dimensional theory responsible for the mixing of different families whose indices were suppressed in the notation for simplicity. We have not included in (2) the leptonic sector nor the $\mathrm{SU}(3)_c$ dynamics because they are not relevant for our proposes. The low-energy theory will only have zero modes for fields that are even under Z_2 symmetry: this is the case for the Higgs doublet that we choose to be even under this symmetry in order to have a standard zero mode Higgs field. The Fourier expansions of the fields are given by

$$B_\mu(x,y) = \frac{1}{\sqrt{\pi R}}B^{(0)}_\mu(x) + \frac{\sqrt{2}}{\pi R}\sum_{n=1}^\infty B^{(n)}_\mu(x)\cos\left(\frac{ny}{R}\right),$$

$$B_5(x,y) = \frac{\sqrt{2}}{\pi R}\sum_{n=1}^\infty B^{(n)}_5(x)\sin\left(\frac{ny}{R}\right),$$

$$Q(x,y) = \frac{1}{\sqrt{\pi R}}Q^{(0)}_{\mathrm{L}}(x)$$
$$+ \frac{\sqrt{2}}{\pi R}\sum_{n=1}^\infty\left[Q^{(n)}_{\mathrm{L}}(x)\cos\left(\frac{ny}{R}\right) + Q^{(n)}_{\mathrm{R}}(x)\sin\left(\frac{ny}{R}\right)\right],$$

$$U(x,y) = \frac{1}{\sqrt{\pi R}}U^{(0)}_{\mathrm{R}}(x)$$
$$+ \frac{\sqrt{2}}{\pi R}\sum_{n=1}^\infty\left[U^{(n)}_{\mathrm{R}}(x)\cos\left(\frac{ny}{R}\right) + U^{(n)}_{\mathrm{L}}(x)\sin\left(\frac{ny}{R}\right)\right],$$

$$D(x,y) = \frac{1}{\sqrt{\pi R}}D^{(0)}_{\mathrm{R}}(x)$$
$$+ \frac{\sqrt{2}}{\pi R}\sum_{n=1}^\infty\left[D^{(n)}_{\mathrm{R}}(x)\cos\left(\frac{ny}{R}\right) + D^{(n)}_{\mathrm{L}}(x)\sin\left(\frac{ny}{R}\right)\right]. \qquad (3)$$

The expansions for B_μ and B_5 are similar to the expansions for the gauge fields and the Higgs doublet (but the latter one without the μ or 5 Lorentz index). It is by integrating the fifth y component in (1) that we obtain the usual interaction terms and the KK spectrum for ED models.

The interaction terms relevant for our calculation will be written in a non-linear R_ξ gauge [18, 65, 66]. For example, in this gauge there is no mixing between the gauge bosons and the charged and neutral unphysical Higgs fields. Besides, the interaction terms are simplified in such a way that there are no trilinear terms such as $W_\mu^+ G_W^- A_\mu$, where G_W^- is an unphysical Higgs field. We are interested in the third family of quarks and $Q_t^{(n)}$ and $Q_b^{(n)}$ are the upper and lower parts of the doublet Q. Similarly, the $U_t^{(n)}$ and $D_b^{(n)}$ are the KK modes of the usual right-handed singlet top and bottom quarks, respectively. There is a mixing between the mass and gauge eigenstates of the KK top quarks ($Q_t^{(n)}$ and $U_t^{(n)}$), where the mixing angle is given by $\tan(2\alpha_t^n) = m_t/m_n$ with $m_n = n/R$. For the b quark the mixing is quite similar, but at leading order the only masses that remain are m_t and m_n, and in this limit the mixing angle is zero. This leads to the spectrum $m_{Q_b^n} = m_n$ and $m_{Q_t^n} = m_{U^n} = \sqrt{m_t^2 + m_n^2}$ for the excited modes of the third family. After dimensional reduction, the fifth components of the charged gauge fields, $W_5^{-(n)}$, mix with the KK modes of the charged component $\phi^{-(n)}$ of the Higgs doublet. The unmixed states are thus the charged physical boson, the excited $G_W^{-(n)}$ state, and a would be Goldstone boson, $\phi_G^{-(n)}$, that contributes to the mass of the KK gauge bosons:

$$\phi_G^{\pm(n)} = \frac{m_n W_5^{\pm(n)} + iM_W \phi^{\pm(n)}}{\sqrt{m_n^2 + M_W^2}} \xrightarrow[M_W \to 0]{n \neq 0} W_5^{\pm(n)},$$

$$G_W^{\pm(n)} = \frac{iM_W W_5^{\pm(n)} + m_n \phi^{\pm(n)}}{\sqrt{m_n^2 + M_W^2}} \xrightarrow[M_W \to 0]{n \neq 0} \phi^{\pm(n)}. \quad (4)$$

The final expression for the Lagrangian can be found in the appendix, where we show the terms that contribute to the decays we are interested in.

3 The $t \to c\gamma$ decay rate

In this section, we present the calculation at one-loop level of the $t \to c\gamma$ process in the framework of a five dimensional universal ED model. We start with a naive calculation comparing the decay widths calculated in the SM and ED model and assuming that in the ED model only the third generation is running in the loop. The one-loop SM width for the top quark decay into a charm quark plus a gauge boson can be approximated by

$$\Gamma(t \to cV) \simeq |V_{bc}|^2 \alpha \alpha_{\text{em}}^2 m_t \left(\frac{m_b}{M_W}\right)^4 \left(1 - \frac{m_V^2}{m_t^2}\right)^2, \quad (5)$$

where for a photon, the neutral gauge boson and a gluon we have $\alpha = \alpha_{\text{em}}$ ($V = \gamma, Z$) or $\alpha = \alpha_{\text{s}}$ ($V = g$), respectively. These results can be compared to the ones expected for extra dimensions, where the ratio m_b/M_W is replaced by M_W/m_n. Using these approximations we can naively estimate the ratio

$$\frac{\Gamma(t \to c\gamma)_{\text{ED}}}{\Gamma(t \to c\gamma)_{\text{SM}}} \simeq \left[\frac{\left(\sum_n (M_W/m_n)^2\right)^2}{(m_b/M_W)^4}\right]. \quad (6)$$

The sum on the KK tower of excited states can be evaluated as we will explain later in the text, and we obtain

$$\frac{\Gamma(t \to c\gamma)_{\text{ED}}}{\Gamma(t \to c\gamma)_{\text{SM}}} = \frac{\pi^4}{36} \left[\frac{(M_W/(1/R))}{(m_b/M_W)}\right]^4 \simeq 1.2 \times 10^2 \quad (7)$$

for $R^{-1} \sim 0.5$ TeV. We have already mentioned that the SM prediction for the branching fraction for the decay $t \to c\gamma$ is of the order $\text{Br}(t \to c\gamma) \sim 10^{-12}$. Then, from (7) the branching fraction for ED models is $\text{Br}(t \to c\gamma)_{\text{ED}} \sim 1 \times 10^{-10}$ for $R^{-1} = 0.5$ TeV.

The naive result on the $\Gamma(t \to c\gamma)_{\text{ED}}$ motivates a complete analysis of the one-loop amplitude in extra dimensions. The general transition $q_i \to q_j + \gamma$ for arbitrary quark flavors i, j in a non-linear R_ξ gauge was studied in [65, 66], where it was found that a reduced number of Feynman diagrams as well as simplified Ward identities greatly facilitate the calculation in this R_ξ gauge.

For on-shell quarks and real photons the transition matrix element is given by

$$M_\mu = i\sigma_{\mu\nu} k^\nu \left(F_2^{\text{L}} m_c P_{\text{L}} + F_2^{\text{R}} m_t P_{\text{R}}\right), \quad (8)$$

where k_μ is the photon momentum, $P_{\text{R,L}} = (1 \pm \gamma_5)/2$, and the magnetic transition form factors $F_2^{\text{L,R}}$ are

$$2F_2^{\text{L}} m_c = (B_1 + B_3 + 2B_5)m_t - A_3 m_c + A_{12} - 2A_{11}, \quad (9)$$

$$F_2^{\text{R}} m_t = (A_1 + A_3 + 2A_5)m_t - B_3 m_c + B_{12} - 2B_{11}, \quad (10)$$

where the A_i and B_i form factors are gotten from the most general Lorentz structure of the renormalized proper vertex [65, 66].

Electromagnetic gauge invariance restricts the amplitude of this decay to the form

$$\mathcal{M}(t \to c\gamma) = \frac{ieg^2}{2^6 c_{\text{W}}^2 \pi^2 m_t^2} \epsilon_\alpha^* k_\mu \left(2\widetilde{F}_2^{\text{R}} m_t\right) \bar{u}_c \sigma^{\alpha\mu} P_{\text{R}} u_t, \quad (11)$$

where c_{W} is the cosine of the Weinberg mixing angle and the form factors F_2^{R} and $\widetilde{F}_2^{\text{R}}$ are related by

$$F_2^{\text{R}} m_t = \frac{eg^2}{32 c_{\text{W}}^2 \pi^2 m_t} \widetilde{F}_2^{\text{R}}. \quad (12)$$

The decay width for this process can be written as

$$\Gamma(t \to c\gamma) = \frac{\alpha^3 m_t}{2^9 \pi^2 s_w^4 c_w^4} \left|\widetilde{F}_2^{\text{R}}\right|^2 \approx 4.8 \times 10^{-7} \left|\widetilde{F}_2^{\text{R}}\right|^2. \quad (13)$$

In order to perform the one-loop calculation, we consider two scenarios. The first one, in which the mass of the excited states associated to the quarks from the three low-energy families are quasi-degenerate at tree level, is without any kind of radiative corrections to the KK masses. In this case, when the excitations coming from the other quarks are taken into account, the transition amplitude for the process $t \to c\gamma$ takes the form

$$\propto \sum_{i=d,s,b} V_{ti}V_{ci}^* \frac{1}{(n/R)^2 + m_i^2}$$
$$\approx V_{tb}V_{cb}^* \frac{1}{(n/R)^2} \frac{m_b^2}{(n/R)^2}, \qquad (14)$$

where the last line can be obtained using the unitarity of the CKM matrix and considering the electroweak corrections of the first two families to be of order zero. m_i is the mass of the down quark running into the loop. Therefore in this scenario we notice that the naive expectation of the decay width $\Gamma(t \to c\gamma)$ given by (7) is suppressed by the factor $(m_b/(n/R))^4$, and then the final result, including the KK states, is smaller than the SM value.

In the second scenario, we consider the most important contribution to the loop correction to come from the excited KK states associated to the third generation. This is a more realistic scenario, because there is a mass hierarchy in the KK states from the different families, such as at low energy. We should mention that in universal extra dimension theories, the fixed points from the orbifold break the translational symmetry of the extra dimension and it is possible to introduce new interactions on the branes. In these new interactions, there are counterterms that cancel the divergences of the radiative corrections, mass terms, and mixing terms from the different family KK modes [67–70]. All our results are presented in the context of this scenario.

Some of the Feynman rules for the model of Sect. 2 can be found in the appendix, where all the relevant terms are shown. In Fig. 1, we illustrate the topology of the one-loop diagrams that are contributing.

In all the decays we are interested in, we neglect corrections of order $(m_c/M_W)^2$. The mixing angles between the gauge and mass eigenstates for the KK excitations of the quarks are written in Sect. 2 and are zero for the leading order approximation. Other possible contributions are neglected due to the Yukawa coupling constants. For example, diagrams involving $G_W^{-(n)} \bar{Q}_b^{(n)} c_R^{(0)}$ are proportional to the Yukawa coupling λ_c and therefore to m_c^2. Diagrams with $G_W^{-(n)} \bar{D}_b^{(n)} t_L^{(0)}$ in the loop are proportional to λ_b^2 and also to m_b^2 and can be neglected.

The leading contributions of type 1 diagrams (see Fig. 1) to the decay come from the following particles circulating in the loop:

$$W_5^{(n)}Q_b^{(n)}Q_b^{(n)}, \qquad W_\mu^{(n)}Q_b^{(n)}Q_b^{(n)},$$
$$G_W^{(n)}Q_b^{(n)}Q_b^{(n)}, \qquad G_W^{(n)}D_b^{(n)}D_b^{(n)}, \qquad (15)$$

where the external photon is coupled to the fermion in the loop. The sum of all these diagrams gives for the form factor F_2^R the following expression:

$$2F_2^R = \frac{2}{3}g^2 e V_{tb}V_{cb}^* \frac{i}{16\pi^2} \int_0^1 dz \int_0^{1-z} dw \frac{1-w-z}{\widetilde{X}}$$
$$\times \left[z + (1-w) + z\frac{m_b^2}{M_W^2} \right], \qquad (16)$$

where the factor $\widetilde{X}$ comes from the dimensional regularization tricks of the product of inverse propagators of the particles circulating in the loop,

$$\widetilde{X} = m_t^2 z^2 + m_t^2 wz - m_t^2 z + m_n^2. \qquad (17)$$

The main contribution of type 2 diagrams (see Fig. 1) is the one with a KK excitation of the standard model gauge boson or a scalar field circulating in the loop, coupled to the external photon:

$$W_\mu^{(n)}W_\mu^{(n)}Q_b^{(n)}, \quad W_5^{(n)}W_5^{(n)}Q_b^{(n)}, \quad G_W^{(n)}G_W^{(n)}D_b^{(n)},$$
$$G_W^{(n)}G_W^{(n)}Q_b^{(n)}, \quad \left(G_W^{(n)}W_\mu^{(n)}Q_b^{(n)} + W_\mu^{(n)}G_W^{(n)}Q_b^{(n)} \right). \qquad (18)$$

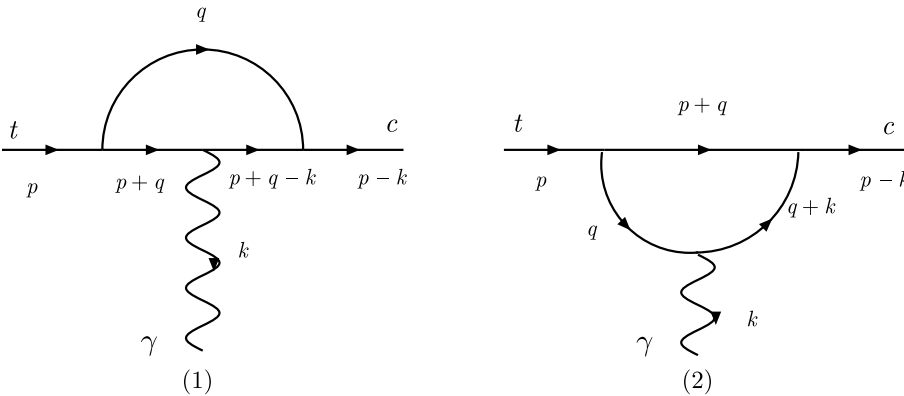These terms contribute to the F_2^R form factor by the following expression:

Fig. 1. Diagrams contributing to the $t \to c\gamma$ decay in the non-linear R_ξ gauge

$$2F_2^{\mathrm{R}} = -g^2 e \frac{\mathrm{i}}{16\pi^2} V_{tb} V_{cb}^* \int_0^1 \mathrm{d}z \int_0^{1-z} \mathrm{d}w$$
$$\times \frac{1}{\widetilde{X}} \left\{ 4(2 - 3w - 4z + 2wz + 2z^2) \right.$$
$$\left. -(1 - w - z)\left[(w + z) - (w + z - 1/2)\frac{m_b^2}{M_W^2} \right] \right\}. \tag{19}$$

For a mass scale of the excited states much higher than the electroweak scale, i.e., $m_n \gg m_t$, the denominator can be approximated by $\widetilde{X} \simeq m_n^2$. We also consider the excited quarks to rotate from interaction to mass eigenstates with the same matrix as that of the ordinary quarks. So, in the Yukawa Lagrangian the interactions $\bar{Q}_{bL} t_{\mathrm{R}}^{(0)} G_W^{-(n)}$ and $\bar{Q}_{bL} c_{\mathrm{R}}^{(0)} G_W^{-(n)}$ are proportional to V_{tb} and V_{cb} respectively, in the mass eigenstates. If we compare the leading contribution coming from these diagrams to the SM contribution [4–6], we find

$$\frac{1}{m_n^2} : \frac{m_b^2}{M_W^2} \frac{1}{M_W^2}. \tag{20}$$

The numerical estimation of all these contributions is straightforward. All the excited mass terms are proportional to n/R, except for the electroweak correction coming from the symmetry breaking. From the numerical point of view this correction does not change the results and can be neglected without modifying the final estimates. Based on these hypotheses, we can also take $\widetilde{X} \simeq m_n^2$, and then the sum over all the KK excited states can easily be done:

$$\sum_n \frac{1}{\widetilde{X}} \simeq \sum_n \frac{1}{\left(\frac{n}{R}\right)^2} = \frac{\pi^2}{6} \frac{1}{\left(\frac{1}{R}\right)^2}, \tag{21}$$

where in any numerical estimate $1/R \simeq \mathcal{O}(1)$ TeV.

Thus, within this approximation, the sum over all the excited KK states is equivalent to multiplication of the results obtained for the first KK excited state by the factor $\pi^2/6$. The sum of all contributions using (16) and (19) gives

$$2F_2^{\mathrm{R}} = \frac{g^2 e}{3} V_{tb} V_{cb}^* \frac{\mathrm{i}}{16\pi^2} \int_0^1 \mathrm{d}z \int_0^{1-z} \mathrm{d}w$$
$$\times \frac{1}{\widetilde{X}} \left\{ -22 + 35w - w^2 + 51z - 30wz - 29z^2 \right.$$
$$\left. + (-4z^2 - 4wz + 10z + 6w + 3)\frac{m_b^2}{2M_W^2} \right\}$$
$$\approx \frac{g^2 e}{18 m_n^2} V_{tb} V_{cb}^* \frac{\mathrm{i}}{16\pi^2} \left\{ -\frac{5}{2} + 11\frac{m_b^2}{M_W^2} \right\}. \tag{22}$$

By using (13) and (12), the numerical value for the decay width is

$$\Gamma(t \to c\gamma) = 1.65 \times 10^{-10}\ \mathrm{GeV}, \tag{23}$$

for $R^{-1} = 0.5$ TeV, and the branching fraction is

$$\mathrm{Br}(t \to c\gamma) \equiv \frac{\Gamma(t \to c\gamma)}{\Gamma(t \to Wb)} = 1.08 \times 10^{-10}. \tag{24}$$

This result shows a branching ratio two orders of magnitude above the SM one.

4 The $t \to cH$ decay rate

The invariant amplitude for the flavor changing decay of a top quark into a charm quark plus a SM Higgs particle can be written as

$$\mathcal{M}(t \to cH) = \bar{u}_c(p)(F_{\mathrm{L}} P_{\mathrm{L}} + F_{\mathrm{R}} P_{\mathrm{R}}) u_t(k), \tag{25}$$

where F_{L} and F_{R} are form factors. In our notation we identify the external scalar Higgs field H with the zero mode Higgs field $h^{(0)}$. From this amplitude we can compute the decay width

$$\Gamma(t \to cH) = \frac{m_t}{32\pi} \left(1 - \frac{m_H^2}{m_t^2} \right)^2 \left(|F_{\mathrm{L}}|^2 + |F_{\mathrm{R}}|^2 \right). \tag{26}$$

The leading diagrams that contribute to the decay are shown in Fig. 2. The leading group of type 1 diagrams in Fig. 2 is the one with KK excitations of the SM quarks circulating in the loop, which are coupled to the external Higgs:

$$W_\mu^{(n)} Q_b^{(n)} D_b^{(n)}, \quad G_W^{(n)} Q_b^{(n)} Q_b^{(n)}, \quad W_5^{(n)} Q_b^{(n)} D_b^{(n)}. \tag{27}$$

In this case, the external Higgs is coupled to the excited quark Q_b generating a flavor changing $\bar{Q}^{(n)} D_b h^{(0)}$ (see the appendix), which is proportional to the bottom quark mass m_b. The contributions to the F_{L} and F_{R} form factors are of the order of zero at leading order, i.e. $F_{\mathrm{L}} = F_{\mathrm{R}} \approx 0$.

The leading diagram of type 2 in Fig. 2 is the one with KK excitations of the standard model gauge bosons and scalar fields circulating in the loop, which are coupled to the external higgs boson $h^{(0)}$:

$$W_5^{(n)} W_5^{(n)} Q_b^{(n)}, \quad G_W^{(n)} G_W^{(n)} Q_b^{(n)}, \quad W_\mu^{(n)} W_\mu^{(n)} Q_b^{(n)},$$
$$W_\mu^{(n)} G_W^{(n)} Q_b^{(n)}, \quad G_W^{(n)} W_\mu^{(n)} Q_b^{(n)},$$
$$\left(W_5^{(n)} G_W^{(n)} Q_b^{(n)} + G_W^{(n)} W_5^{(n)} Q_b^{(n)} \right), \tag{28}$$

and these contribute to the form factor by the following expressions:

$$F_{\mathrm{L}} = g^3 V_{tb} V_{cb}^* \frac{\mathrm{i}}{16\pi^2} \int_0^1 \mathrm{d}z \int_0^{1-z} \mathrm{d}w \frac{1}{\widetilde{X}} \left\{ 2(1-w)m_t M_W \right.$$
$$\left. - \left[(z-1)(w+z)m_t^2 + w(z-2)m_h^2 \right] \frac{m_t}{m_W} \right\},$$
$$F_{\mathrm{R}} = g^3 V_{tb} V_{cb}^* \frac{\mathrm{i}}{16\pi^2} \int_0^1 \mathrm{d}z \int_0^{1-z} \mathrm{d}w \frac{1}{\widetilde{X}} \{ w m_t M_W \}. \tag{29}$$

After evaluating the parametric integrals, the form factors are given by

$$F_{\mathrm{L}} = \frac{g^3}{6 m_n^2} V_{tb} V_{cb}^* \frac{\mathrm{i}}{16\pi^2} \left\{ 8 M_W m_t + \frac{5 m_t^3}{M_W} + \frac{7 m_h^2 m_t}{2 m_W} \right\},$$
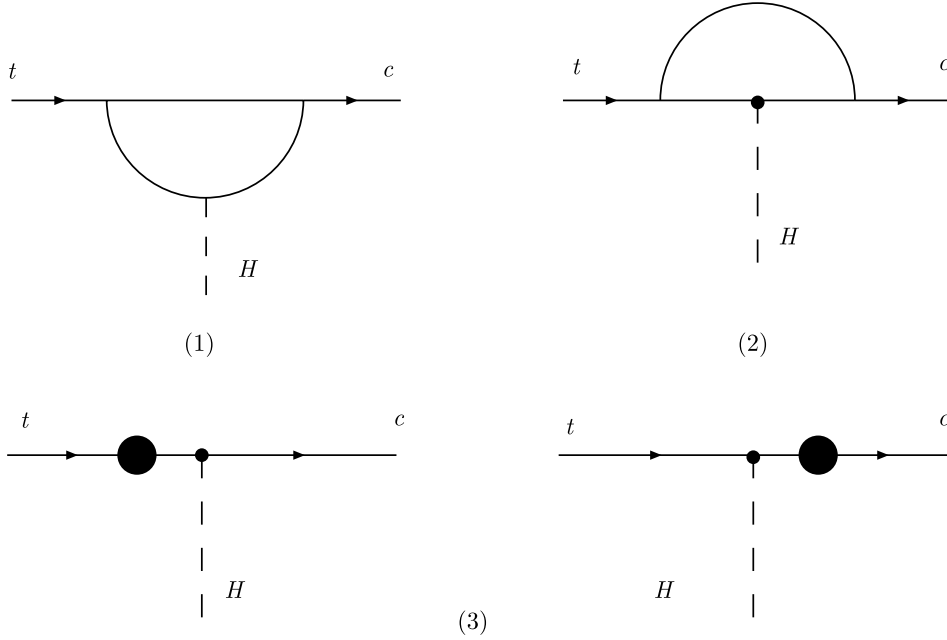$$F_{\mathrm{R}} = \frac{g^3}{6 m_n^2} V_{tb} V_{cb}^* \frac{\mathrm{i}}{16\pi^2} \{ M_W m_t \}. \tag{30}$$

Fig. 2. Feynman diagrams for the $t \to cH$ decay in extra dimensions

Finally, the type 3 diagrams in Fig. 2 coming from a renormalized flavor changing fermion line do not contribute at leading order to the decay width. The first one of these diagrams is proportional to the charm quark mass, because of the Higgs coupling, and therefore is negligible. For a similar reason, the second diagram has a top quark mass factor, but the self-energy part introduces the charm quark mass and this contribution is suppressed with respect to the leading order.

From these form factors and (26), we compute the $t \to cH$ decay width. Finally, the branching ratio is $\mathrm{Br}(t \to cH) = 1.08 \times 10^{-10}$, for $m_H = 120\,\mathrm{GeV}$.

5 Conclusions

We have computed the decay widths $\Gamma(t \to c\gamma)$ and $\Gamma(t \to cH)$ in a universal extra dimension model with a single extra dimension, where we have considered the most important contribution to the loop correction to come from the excited KK states associated to the third generation. The results show a branching ratio that is above the SM one. The branching ratios for these two decay widths are of the order of 10^{-10}.

There is a strong dependence on the top quark mass m_t in the amplitude of the $t \to cH$ process; it comes from the type 2 diagrams in Fig. 2 with an excited scalar in the loop, resulting in a m_t^3/M_W^3 factor. When we take the limit $m_n \gg m_t$, we find that the decay widths for the $t \to c\gamma, (H)$ processes are decoupled with respect to the new scale $1/R$, and they go to zero. Considering the excited states of the quarks to be quasi-degenerate and the CKM matrix to be unitary, the amplitudes for the flavor changing decay t–c due to the contribution of the excited states of the quarks are suppressed by the factor $(m_b/(n/R))^2$, and the predicted values are smaller than the SM predictions.

Appendix

The Lagrangian can be separated in different terms as in the following sum:

$$L = \sum_{m=1,2,3} \sum_{n=1}^{\infty} L_m^{(n)}. \qquad (A.1)$$

After symmetry breaking the interaction terms are included in the terms $L_1^{(n)}$ up to $L_3^{(n)}$. The first one, $L_1^{(n)}$, is

$$L_1^{(n)} = \frac{gm_t}{\sqrt{2}M_W} \bar{Q}_{bL}^{(n)} t_R^{(0)} G_W^{-(n)}$$
$$- \frac{gm_b}{\sqrt{2}M_W} \bar{D}_{bR}^{(n)} t_L^{(0)} G_W^{-(n)} + \mathrm{h.c.} \qquad (A.2)$$

The second term, $L_2^{(n)}$, has the excited gauge boson–fermion interactions

$$L_2^{(n)} = \frac{2}{3} e A_\mu^{(0)} \left[\bar{Q}_t^{(n)} \gamma^\mu Q_t^{(n)} + \bar{u}_t^{(n)} \gamma_\mu \bar{u}_t^{(n)} \right]$$
$$+ \left\{ \frac{g}{\sqrt{2}} W_\mu^{+(n)} \bar{Q}_{bL}^{(n)} \gamma^\mu t_L^{(0)} \right.$$
$$+ \mathrm{i} e A_\mu^{(0)} \left[G_W^{-(n)} \partial^\mu G_W^{+(n)} + W_5^{-(n)} \partial^\mu W_5^{+(n)} \right]$$
$$+ \frac{g}{\sqrt{2}} \bar{Q}_{bR}^{(n)} t_L^{(0)} W_5^{-(n)}$$
$$\left. + 2e \frac{n}{R} A_\mu^{(0)} \left[W_\mu^{-(n)} W_5^{+(n)} - W_\mu^{+(n)} W_5^{-(n)} \right] + \mathrm{h.c.} \right\}. \qquad (A.3)$$

And the third term has the neutral Higgs boson interactions:

$$
\begin{aligned}
L_3^{(n)} = & -\frac{gm_t}{2M_W}\bar{Q}_t^{(n)}U_t^{(n)}h^{(0)} - \frac{gm_b}{2M_W}\bar{Q}_b^{(n)}D_b^{(n)}h^{(0)} \\
& + gM_W W_5^{+(n)}W_5^{-(n)}h^{(0)} + gM_W W_\mu^{+(n)}W_\mu^{-(n)}h^{(0)} \\
& - \frac{gm_h^2}{2M_W}G_W^{+(n)}G_W^{-(n)}h^{(0)} \\
& + \left\{ \mathrm{i}\frac{g}{2}\frac{n}{R}W_5^{(n)}G_W^{+(n)}h^{(0)} + \mathrm{i}\frac{g}{2}W_\mu^{+(n)}h^{(0)}\partial^\mu G_W^{-(n)} \right. \\
& \left. + \mathrm{h.c.} \right\} .
\end{aligned}
\tag{A.4}
$$

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